Numerical Study on Flow Separation of A Transonic Cascade

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Objective:

- Numerical prediction of the boundary layer separation for a transonic cascade

Background:

- Flow separation is one of the unsteady aerodynamic forcing sources exciting blade flutter

Motivation:

- Develop a CFD solver to predict the steady and unsteady flow separation for compressor
Governing Equations

3D Reynolds averaged NS equations:

\[
\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} + \frac{\partial G}{\partial z} = \frac{\partial R}{\partial x} + \frac{\partial S}{\partial y} + \frac{\partial T}{\partial z}
\]

where

\[
Q = [\bar{\rho}, \bar{\rho}\tilde{u}, \bar{\rho}\tilde{v}, \bar{\rho}\tilde{w}, \bar{\rho}\tilde{e}]^T \\
E = [\bar{\rho}\tilde{u}, \tilde{p} + \bar{\rho}\tilde{u}^2, \bar{\rho}\tilde{u}\tilde{v}, \bar{\rho}\tilde{u}\tilde{w}, (\bar{\rho}\tilde{e} + \tilde{p})\tilde{u}]^T \\
F = [\bar{\rho}\tilde{v}, \bar{\rho}\tilde{u}\tilde{v}, \tilde{p} + \bar{\rho}\tilde{v}^2, \bar{\rho}\tilde{v}\tilde{w}, (\bar{\rho}\tilde{e} + \tilde{p})\tilde{v}]^T \\
G = [\bar{\rho}\tilde{w}, \bar{\rho}\tilde{u}\tilde{w}, \bar{\rho}\tilde{v}\tilde{w}, \tilde{p} + \bar{\rho}\tilde{w}^2, (\bar{\rho}\tilde{e} + \tilde{p})\tilde{w}]^T \\
R = \frac{1}{Re}[0, \bar{\tau}_{xx}, \bar{\tau}_{xy}, \bar{\tau}_{xz}, \beta_x]^T \\
S = \frac{1}{Re}[0, \bar{\tau}_{xy}, \bar{\tau}_{yy}, \bar{\tau}_{yz}, \beta_y]^T \\
T = \frac{1}{Re}[0, \bar{\tau}_{xz}, \bar{\tau}_{yz}, \bar{\tau}_{zz}, \beta_z]^T
\]

shear stresses are expressed as,

\[
\bar{\tau}_{ij} = -\frac{2}{3}(\tilde{\mu} + \mu_t)\frac{\partial \tilde{u}_j}{\partial x_i} + (\tilde{\mu} + \mu_t)\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right)
\]

\[
\beta_x, \beta_y, \text{ and } \beta_z \text{ are expressed as,}
\]

\[
\beta_i = (\tilde{\mu} + \mu_t)\tilde{u}_j\tau_{ij} + \frac{1}{\gamma - 1}\left(\frac{\tilde{\mu}}{Pr} + \frac{\mu_t}{Pr_t}\right)\partial \tilde{a}^2
\]
Numerical Algorithms

- Implicit Gauss-Seidel Relaxation Time Marching

- Roe and Van Leer Schemes for inviscid fluxes, 3rd Order MUSCL-type differencing

- 2nd order central differencing for viscous terms

- Baldwin-Lomax Turbulence model
Figure 1: Computed velocity profile comparison with the law of the wall
Figure 2: *The transonic inlet-diffuser mesh*

Figure 3: *Mach number contours of the transonic inlet-diffuser*
Figure 4: Upper wall pressure distribution of the transonic inlet-diffuser
Figure 5: NASA transonic flutter cascade tunnel
Figure 6: **Cascade 3D mesh**
Figure 7: Cascade 3D mesh
Figure 8: Flow pattern of the inlet-diffuser at incidence
Figure 9: Mid-span static pressure distribution at Mach number 0.5
Figure 10: Mid-span fbw pattern under different inlet Mach numbers

(a) Ma=0.5  
(b) Ma=0.8  
(c) Ma=1.18
Figure 11: Suction surface fbw pattern at Mach number 0.5
Figure 12: Suction surface fbw pattern at Mach number 0.8
Figure 13: Suction surface fbw pattern at Mach number 1.18
Figure 14: Mid-span static pressure distribution at Mach number 0.5
Figure 15: Mid-span static pressure distribution at Mach number 0.8
Figure 16: Experimental shock structure of the NASA transonic cascade at Mach number 1.18
Conclusions:

- NASA GRD flutter compressor cascade calculated at incidence of $0^\circ$ and $10^\circ$, inlet $M=0.5, 0.8, 1.0$

- The surface pressure distribution agrees well with the experiment with no flow separation.

- At high incidence and subsonic, flow separation starts at leading edge.

- The separation bubble length predicted agree well with the experiment.

- The computed surface pressure with separation rises more steeply than that at experiment, overall agreement is reasonable.

- At supersonic, the flow is attached-separated-reattached. The separation is due to the shock wave/boundary layer interaction.

- With Baldwin-Lomax turbulence model, the Van Leer scheme predicts the separation region agreeing better than the Roe scheme.